

Year 12 Mathematics Specialist 2019 Test Number 6: Statistical Inference Resource Rich

Teacher: Mrs Da Cruz

Name: _____

Marks: 46

Time Allowed: 45 minutes

Instructions: You are permitted 1 A4 page of notes and your calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1

A Company is asked to make bolts with length accurate to within ± 0.002 with a 95% confidence level. If $\sigma = 0.004$, determine how many bolt lengths must be averaged to satisfy the request.

Question 2

A valid 90% confidence interval, for a sample of 10 observations based on a population standard deviation of 0.05 with a $\bar{x} = 0.927$, is 0.901 to 0.953.

a) What must be true about the population?

b) Describe the effect on the width of the confidence <u>interval</u> when the confidence <u>level</u> is increased. Justify your answer with some calculations.

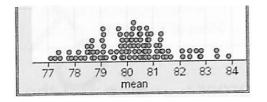


[2 marks]

[3 marks]

Question 3

Using simulation, 100 random samples, each of size 49, is generated and the sample means are shown in the dot plot.



- a) Indicate whether the following statements are true or false.
- b) For any of the statements which are false, clearly explain why they are false.
 - i. As the dot plot appears to show that the sampling distribution of the sample means is normally distributed, the population must have been normal. True / False
 - ii. The dot plot appears to show that the sampling distribution of sample means is normal with a mean of approximately 80 and standard deviation of approximately 1. True / False
 - iii. The information in the dot plot implies that the population mean was approximately 80 with a standard deviation of approximately 1. True / False
 - iv. The information in the dot plot implies that the population mean was approximately 80 with a standard deviation of approximately 7. True / False
 - v. The information in the dot plot shows a sample which is normally distributed, mimicking the population distribution. True / False

Question 4

The mass of crayfish caught near the Abrolhos Islands is observed to be normally distributed with a mean of $\mu = 1.2$ kg and standard deviation of $\sigma = 0.25$ kg.

Joe the fisherman catches 65 crayfish.

- (a) Determine the probability that:
 - (i) the mean crayfish mass will be less than 1.15 kg. (3 marks)

(ii) the total mass will be between 75 kg and 80 kg. (3 marks)

On another fishing trip, we are required to be 98% confident that the mean crayfish mass differs from the population mean by less than 0.05 kg.

(b) Find the number of crayfish that need to be caught. (2 marks)

A rival crayfisherman, Jamie, has started catching crayfish further out to sea than Joe. Jamie states that the crayfish caught are significantly bigger than in the area that Joe fishes in.

Over a month Jamie catches 220 crayfish with total mass of 270 kg. Assume $\sigma = 0.25$ kg.

(c) Determine whether Jamie's claim is supported at the 95% level of confidence. (4 marks)

Question 5 [16 marks]

The volume of water used by the SavaDaWater company to top up an ornamental pool has been observed to be normally distributed with mean $\mu = 175$ litres and standard deviation $\sigma = 15$ litres.

The ornamental pool is topped up 50 times. Determine the probability that the:

(a) sample mean volume will be between 173 and 177 litres. (3 marks)

(b) total volume of water used is less than 8.96 kilolitres. (3 marks)

Water is a scarce commodity and accuracy is required. The pool is topped up 50 times and the sample mean obtained is denoted by \overline{W} .

(c) If it is required that $P(a \le \overline{W} \le b) = 0.99$, then determine the values of *a* and *b*, each correct to 0.1 litres. (3 marks)

(d) If the probability for the mean amount of water used differs from μ by less than five litres is 96%, find *n*, the number of waterings that need to be measured. (3 marks)

A rival company called WolliWorks takes over the watering of the ornamental pool. Over 36 consecutive days, it was observed that the WolliWorks company used a total of 6.57 kilolitres. The standard deviation for the 36 days was also 15 litres.

A representative from the SavaDaWater company states that 'WolliWorks are using significantly more water than we did when we were filling this pool. They are wasting water'.

(e) Perform the calculations necessary to comment on this claim. (4 marks)

Question 6 [5 marks]

a. Simulate a sample of 80 items from a uniform distribution on the interval [20,40] and sketch the graph of **your** sample. (1 mark)

- b. Simulate 200 sample of 80 items from a uniform distribution on the interval [20,40] and find the mean of the samples. (1 mark)
- c. Draw a graph of the sampling distribution.

(1 mark)

d. Comment on the graphs.

(2 marks)